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Axioms and Their Validation in Aristotle and Ayn Rand

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> [Aristotle] has been opposed, misinterpreted, misrepresented, and—like an axiom—used by his enemies in the very act of denying him.
> AYN RAND, *THE VOICE OF REASON*

Ayn Rand's novel *Atlas Shrugged* portrays a world in collapse, a collapse being aided and hastened by the refusal of a number of key intellectuals, artists, inventors, and industrialists to contribute in any way to the irrationalism and exploitation sweeping the world. Among those on strike is a young man named Ragnar Danneskjöld, who majored in physics and philosophy in college and is preparing for a career teaching philosophy, but who emerges early in the novel as a modern-day pirate on the high seas, mysteriously restricting his piracy to ships carrying cargo paid for with government-expropriated wealth. Those who are interested in finding out why will have to read the novel. I mention this particular character here because of Ayn Rand's portrayal of him in a scene in the closing pages of *Atlas Shrugged*:

> Ragnar Danneskjöld lay stretched out on a couch, reading a volume of the works of Aristotle: " . . . for these truths hold good for everything that is, and not for some special genus apart from others. And all men use them, because they are true of being *qua* being. . . . For a principle which everyone must have who understands anything that is, is not a hypothesis. . . . Evidently then such a principle is the most certain of all; which

> principle this is, let us proceed to say. It is, that the same attribute cannot at the same time belong and not belong to the same subject in the same respect." (*Atlas* 1167)

Danneskjöld is reading excerpts from the W. D. Ross translation of *Metaphysics* Γ.3, where Aristotle identifies the axiomatic status of the Principle of Non-Contradiction (PNC). Rand's decision to quote Aristotle in the closing scene of her magnum opus, and to select that quotation in particular, reveals a great deal both about her approach to philosophy and about her understanding of the relationship between her philosophy and Aristotle's.

Nor is this passage that novel's most obvious reference to the axioms of Aristotelian logic. *Atlas Shrugged* is divided into three parts, the titles of which are: "Non-Contradiction," "Either-Or," and "A Is A." As she acknowledged in the "About the Author" page at the end of the first edition: "[Aristotle's] definition of the laws of logic and of the means of human knowledge is so great an achievement that his errors are irrelevant by comparison" ("About the Author," *Atlas*).[1]

Although intended in part as an acknowledgment of Aristotle's achievement, these headings are also a key to the main themes of the novel's three parts. The first section is dominated by what appears to the two central characters in that part, Dagny Taggart and Hank Rearden, to be a contradiction[2]—those who have created America's greatest art, science, philosophy, and industry are abandoning it to its destroyers. At key moments they are challenged to question this apparent contradiction—on the grounds that, in reality, there are no contradictions. In the second section of the book a stark choice is revealed between two, diametrically opposed, philosophical approaches to the world, and it becomes increas-

1. It is important to note that in *Metaphysics* Γ Aristotle only discusses the Principles of Non-Contradiction and Excluded Middle. In an unpublished part of the Epistemology Workshops (Workshops 272), Rand, while answering a question about the relation of mathematical axioms to the Law of Identity, expresses her understanding of the relationship between it and these two principles: "I think the relationship is like between the law of identity and the other two laws of logic, which are only corollary restatements but they're restatements that are very valuable, epistemologically, as guidelines. But they in fact say the same thing" (see also Peikoff 1991, 119). I will return to the question of whether Aristotle recognizes, or comes close to recognizing, this point. But in *Atlas Shrugged*, Rand credits Aristotle with stating the Principle of Identity: "Centuries ago, the man who was—no matter what his errors—the greatest of your philosophers, has stated the formula defining the concept of existence and the rule of all knowledge: *A is A*. A thing is itself" (*Atlas* 1016).

2. Actually, many apparent contradictions. For an excellent discussion of them, see Ghate (2009, 2–19).

ingly clear that it is 'either-or.' Finally, in the last section, the culture dominated by one of those philosophical approaches is, as is inevitable, collapsing; and we are introduced to a hidden valley populated by people who live by the other philosophical approach and who are at peace with the Principle of Identity. Ragnar Danneskjöld is one of those people, and he is reading that passage from *Metaphysics* Γ while residing in that valley.

Ayn Rand thus pays tribute to Aristotle's identification and validation of fundamental axioms as a primary task of metaphysics, the science of being qua being, as one of his greatest philosophical achievements. Nevertheless, Ayn Rand's philosophical discussions of fundamental axioms are importantly different from Aristotle's. In the next two sections of this paper I will present their views on this topic somewhat independently, with respect to three distinct questions: (1) Reference: To what do axioms refer? (2) Function: What is their cognitive function? and (3) Validation: Can axioms be validated, and if so, how? In the third and fourth sections, I explore their similarities and differences on these three issues. Two conclusions will become clear in the process: first, Ayn Rand's position on this topic is fundamentally Aristotelian; and second, what is distinctive in her approach reflects distinctive and innovative features of her epistemology.

Aristotle on Axioms and Their Validation

It is implicit in the very first sentence of the *Posterior Analytics* that Aristotle's account of knowledge in the strict sense, *epistēmē haplōs*, will require some form of indemonstrable starting points. In the very first chapter he cites the Principle of Excluded Middle (PEM)—that "in every case either the affirmation or the denial is true" (*APo.* I.1 71a12–14)—as one such starting point. In chapter 2 he distinguishes two types of immediate starting points: posits and axioms. Of the latter, he notes that anyone who is to learn anything whatsoever must grasp (*echein*) them—and he feels it necessary to add, "for there are some things of this sort" (72a16–17).

The Unsatisfying Analytics *Answer*

These chapters make it clear, then, that there must be *indemonstrable* starting points of demonstration; the only validation for knowledge discussed in these chapters, however, is *demonstration*. How, then, are these indemonstrable starting points of demonstration to be validated? In *Analytics* I.3, Aristotle states this problem in the form of a dilemma: all scientific knowledge proceeds by demonstration; but demonstrations pro-

ceed from primitive and immediate starting points. Either our certainty about these rests on their being demonstrated, in which case they are not primitive and a regress looms (compare *Met.* Γ.4 1006a8–10); or it does not, in which case our alleged demonstrative knowledge rests on starting points that have not been justified (*APo.* I.3 72b8–12). Aristotle admits the validity of the challenge, but his only attempt at a reply here is to assert that there is "not only scientific knowledge (*epistēmē*) but a starting point (*archē*) of scientific knowledge as well, by which we come to know the definitions (*horoi*)" (72b23–25).[3] The axioms, however, are *not* definitions, and so one might wonder about whether this starting point of knowledge (later identified as reason [*nous*; see *APo.* I.33 88b33–37, II.19 100b15]) is intended to be involved in grasping axioms as well as definitions. However one answers that question, the objection Aristotle is facing here is global and raises identical concerns regarding the validation of common axioms.[4]

Later in *Posterior Analytics* I.11, Aristotle discusses the logical axioms and to some extent clarifies their status. He argues that it is only in demonstrations of substitution instances of the PNC that the principle needs to be stated as a premise, and that it is only in the case of a *reductio* that it is necessary to posit PEM explicitly. PNC and PEM are not primarily *archai* in the sense of undemonstrated, primitive *premises* of demonstrations.[5] Rather, they are presuppositions of demonstration itself. Syllogistic inference presupposes them; remove these starting points and anything (and therefore nothing) goes. But in neither chapter does Aristotle face the question of how we come to know these starting points of demonstration, nor what grounds we have for believing in them or in their axiom-

3. See also *Metaphysics* Γ.6 1011a12–13: "But as we said, this is what happens to them; for they search for an argument for things for which there is no argument. For the starting point of demonstration is not a demonstration."

4. The term *horos* here might have the sense it does at *Parts of Animals*: "for the enquiry into nature, too, there should be certain standards (*horoi*), such that by referring to them one can appraise the manner of its proofs, apart from the question of what the truth is" (*PA* I.1 639b13–16). The claim in *PA* I.1 is restricted to natural enquiry, but the characterization of the principles under consideration—standards by which to appraise the manner of proof, independently of considerations of the truth—also applies to PNC and PEM.

5. Some scholars such as Ross (1949, 531) and J. Lear (1980, 101–2) have made much of the terminological distinction between *ek* and *dia* at *APo.* I.11 76b10; but as Mignucci (1975, 141–43) and Barnes (1993, 139) have pointed out, Aristotle will use the preposition *ek* to discuss reasoning in accordance with the axioms and will even, rarely, refer to a common axiom as a *protasis* (premise). Less important than the terminology is the explicit claim of the *Analytics* that these axioms stand in a different relationship to demonstration than do, for example, the definitions and postulates of a science.

atic status.

Aristotle appears to address the problem of the validation of demonstrative starting points, including the logical axioms, in the final chapter (*APo.* II.19). The principles of both the crafts and the sciences, he explains, arise through a process that begins in perception and that involves the retention in memory of perceptions. Human beings have the ability to grasp something common ("whatever is present as one and the same in all of them") by means of comparing these many retained perceptions. We thus derive first level universals directly from perception: "for though one perceives the particular, perception is of the universal—of man, for instance, not of Callias the man" (*APo.* II.19 100a17–b1). Aristotle thus argues that "we come to know the primaries (*ta prōta*) by induction; for that is in fact how perception produces the universal in us" (100b4–6; *APr.* I.30, 46a17–27). The mental state that grasps the starting points of demonstration is, apparently, reason (*nous*), which is a starting point of scientific knowledge in a parallel sense to what it grasps being the starting point of demonstration, and induction is apparently its method.

Regrettably, Aristotle does not here distinguish among kinds of starting points, and the only example he gives in the relevant section of his chapter *APo.* II.19 describes a progression from the perception of particular entities such as individual men to a grasp of wider and wider universals (man, animal) under which they fall. The only reason to think that what is said here has any relevance to our grasp of common axioms such as PNC is the generality of Aristotle's conclusion. He appears to be providing an account of how, and by means of what cognitive faculty, we grasp *all* the starting points of demonstration.[6]

As far as I can see, this question is left without a satisfactory answer in the *Analytics.* This may be because Aristotle has an answer but recognizes it as a topic for another subject domain.[7] On the other hand, it may be (as Irwin tells it) that Aristotle realizes that the *Analytics* is, at least on this topic, a failed epistemological enterprise, and he takes a second shot at an answer, via "strong dialectic" in the *Metaphysics*.[8]

6. On the many different referents of *archē* in the *Analytics*, see Ferejohn 1991, 31–36; on *APo.* II.19 as providing the ultimate "justification" for PNC, see Bolton (1994, 351–54); against which see Irwin (1988, 179–98).

7. There is a hint in this direction in the *Analytics*: "And dialectic associates with all the sciences; and so would any [science] which attempted to give universal proofs of the common [axioms] (e.g. that everything is either asserted or denied, or that equals from equals leave equals, or any other [common axioms] of this sort)" (*APo.* I.11 77a29–31). Since this hypothetical discipline is being contrasted with dialectic, it cannot be dialectic that he has in mind.

8. See Irwin 1988, chs. 7–8. *APo.* I.11 also has hints pointing in this direction.

The More Satisfying Metaphysics *Γ Answer*

The Question of Reference

From the opening chapters of *Metaphysics* A, Aristotle is sending signals that he is on a quest for a new kind of science.[9] And the second puzzle (*aporia*) raised for consideration of this new kind of science in *Metaphysics* B is "whether it is for this science only to contemplate the first principles (*tas archas tas prōtas*) of substantial being (*tēs ousias*), or *also* [for it] to be concerned with the principles on the basis of which everyone proves, e.g. whether it is possible to assert and deny one and the same thing simultaneously or not, and about other such things" (*Met.* B.1 995b7–11). And when he takes up this puzzle for investigation at 996b27, he is explicit about these principles of proof: "I mean by demonstrative [principles] those common beliefs from which all [sciences] prove, e.g. that in all cases it is necessary either to assert or deny, and that it is impossible that something both be and not be simultaneously, and any other such premises: Is there a single science of these and of substantial being, or is there a different science [for each]; and if there is not a single science of both, which ought to be designated the one we are now seeking?" (*Met.* B.2 996b28–33). Why, one might ask, would this question even occur to Aristotle? Why would a science that investigates substantial being also investigate the sorts of postulates that mathematicians refer to as axioms?

It is an odd question for anyone inclined to see statements like the PNC as "laws of logic" or "laws of thought." But let us put this question into Aristotle's context. Parmenides (as quoted in Plato's *Sophist*, 237a, 258d) had proclaimed "Never will this prevail, that what is not is—bar your thought from that road of inquiry"—and drew from that proclamation the implication that the world of change familiar to us by perception must be illusory. Under the influence of Parmenides, Plato, in the *Republic*, has Socrates describe perceptible objects as "dualizing," neither "being" nor "not-being" (*Republic* V 479c3–4, 478e1–5), and draws from this the conclusion that such things cannot be objects of knowledge.

The philosophical world into which Aristotle enters when he joins the Academy is a world locked in a long-standing debate about appearance and reality, to which Aristotle proposes a radical solution. He insists that the senses provide us a direct awareness of being, and in so doing he

9. See also *Met.* A.2 982a5, b4, b8, 983a5, a21; *Met.* B.1 995a24, 2 996b3; and Broadie 2012, 51–55.

rejects the consensus epistemology of "appearance versus reality," a consensus for which his teacher Plato had become the most powerful exponent. When Aristotle opens his *Metaphysics* citing the delight humans take in their sensory awareness as evidence for his claim that all men by nature desire to know, he is striking out in a new direction. In his opening chapter he describes a continuous path to scientific knowledge that begins with perceptual awareness of particular substantial beings, a form of awareness we share with many other animals and which gives us the most accurate knowledge of particulars.

But scientific knowledge depends on demonstration, and demonstration depends on indemonstrable first principles. Many of those will take the form of basic concepts and their definitions, of course; but the very practice of demonstration presupposes that the propositions that make up their premises take one—*and only one*—side of a contradiction.[10] It cannot be the case that swimming simultaneously both belongs and does not belong to whales, or that 2R simultaneously belongs and does not belong to triangles.

In the first three (or four) books of our *Metaphysics*, I suggest, Aristotle is striving to distinguish inquiry into these sorts of issues, a special kind of wisdom that is primary over others because the "special sciences" presuppose it—in his terms, primary philosophy.[11] And he thus sees a defense of these axioms as part of an inquiry into the fundamental nature of reality—that is, as a proper subject for the science of being qua being.

But why would this science be the same science that investigates substantial being? After a long consideration of the fourteen puzzles (*aporiai*) about the subject at hand in *Met.* B, Aristotle opens *Met.* Γ by boldly asserting his answer to the previous book's first puzzle. "There is a science that studies being qua being and those things that belong to it in virtue of what it is" (*Met.* Γ.1 1003a21–22). He explains that it is a science not by virtue of there being a single genus that it investigates, but because there is a

10. "A proposition is one part of a contradictory pair, one thing said of one thing. It is dialectical if it assumes either part indifferently and demonstrative if it determinately assumes one part because it is true. A statement is one part of a contradictory pair. A contradictory pair is a pair between which, in their own right, there is nothing. The part of a contradictory pair which says something of something is an affirmation; the part which takes something from something is a negation" (Barnes translation, slightly modified, *APo.* I.2 72a8–14). There are a number of textual and interpretive issues raised by this passage that are not important here; on which see Barnes 1993, 98–99. There is important background to the distinctions drawn in this passage at *De Interpretation* 6–9.

11. For more detail on this topic, see the essays on *Met.* A.1 (by Cambiano) and *Met.* A.2 (by Broadie) in Steel (2012, 1–67).

primary category of being, substantial being (*ousia*), in relation to which everything else is said to be. One who studies being qua being, then, will focus primarily on the principles and causes of substantial being (*ousia*) (*Met.* Γ.2 1003a33–b23).[12] An interesting question to explore (on another occasion) is exactly what the relationship is between these principles and causes and the most secure truths that Aristotle refers to as common axioms. But this much is clear: an investigation into the topic of whether a being can simultaneously be and not be what it is, and into the principles and causes of substantial being are, in the context of fifth–fourth century Greek philosophy, intimately related investigations.

Metaphysics Γ.3 opens by reminding us of the *second* puzzle of *Met.* B: is it for one and the same science to discuss both substantial being and what the mathematicians call axioms, or are these two topics for two different sciences?[13] The answer, Aristotle claims, is now "apparent": "It is apparent that the investigation of these [*common axioms*] is also for a single science, that is, the science of the philosopher; for they belong to all beings and not to some distinct kind separate from others" (*Met.* Γ.3 1005a21–23).[14] And though he is now clear on this not being an appropriate subject for *Analytics*, he reiterates what he has asserted about the common axioms in *APo.* I.1–2: these principles are the most secure of all, they are non-hypothetical, and the student of any subject at all must come to his subject already knowing these principles (*Met.* Γ.3 1005b5–6, b13–17).

Finally, at *Metaphysics* Γ.3, 1005b18ff., the passage that Ragnar Danneskjöld is reading near the close of *Atlas Shrugged*, Aristotle gets around to identifying the most secure of axioms: "It is impossible for the same thing to belong and not belong to the same thing at the same time and in the same respect (and let as many other determinations as need to be

12. This topic is explored in more detail in Rheins's chapter, "The Subject of Ontology: Aristotle on the Focally Related Meanings of 'Being' and the Univocity of Rand's Axiomatic Concept of 'Existence,'" Chapter 3, below. See also *Met.* Z.1 1028b2–7: "Indeed, the question that was, is, and always will be asked and always will be puzzling, 'What is being?' (*ti to on*) is this question: 'What is substantial being?' (*tis tēs ousias*). For it is this that some say to be one and others more than one, and some say is limited and others unlimited; wherefore the greatest, first and essentially only thing for us to study is what is being in this sense."

13. For reasons of space I am omitting discussion of *Metaphysics* Γ.2, though when read against the backdrop of Plato's discussion of *megista genē* in the *Sophist*, it is of direct relevance to our topic.

14. See also *Met.* Γ.3 1005a27–29: "So since it is clear that they belong to all things qua being (for this is common to them), the study of these things too is for the one acquiring knowledge of being qua being."

added to deal with dialectical objections)" (*Met.* Γ.3 1005b19–21). The PNC (his focus in chs. 3–6) and the PEM (discussed in ch. 7) are universally applicable *because*:

- They "belong to all beings" (*Met.* Γ.3 1005a23)—that is, they identify facts true of all being
- They are used by everyone "because they are of being qua being, and each kind is a being" (1005a24–25)
- They are thus to be studied by the investigator of being qua being "because they belong to all things qua being" (1005a27–28)

Aristotle's fundamental argument for these axioms being investigated by the first philosopher is, then, not epistemic or logical but *metaphysical*. They are truths about all beings, just in so far as they are beings.

Thus, on the Reference Question, Aristotle and Rand agree: the common axioms are not conventions nor do they refer primarily to laws of thought: they identify fundamental aspects of reality, undeniable truths about being qua being.

The Question of Function

One of Aristotle's key tasks in *Metaphysics* Γ.3 is to identify five *epistemic properties* of PNC as a basic truth about being qua being:

1. It is the most firm, secure, or certain (*bebaiotatē*) of all starting points: (1005b10–12; see also 1005b17, 1011b13–15).
2. It is a principle about which it is impossible to be deceived (1005b13).
3. It is best known and non-hypothetical (1005b13–15).
4. It is a truth one must possess to understand anything about things that are (1005b15–18; recall *APo.* I.2 72a16–17).
5. And finally: "it is by its very nature an origin of all the other axioms" (*Met.* Γ.3 1005b33–34; likely he is thinking of mathematics).

That is, he aims to establish that the PNC provides the firmest possible foundation for reasoning of any kind—that is its function. But in order to play this role we have to be certain that it is best known and most secure; and common axioms have a distinctive problem in this respect, which is that all proof depends on them. They cannot be secured by demonstration.

Aristotle has provided, then, an explicit identification of a fundamental truth about being that all of us know implicitly, and he has made a number of second order or philosophical claims about its *epistemic func-*

tion as the most firm and certain foundation of knowledge. In the following chapter he appears to face up to the problem that was left unresolved in the *Posterior Analytics*: its validation.

The Question of Validation

In *Metaphysics* Γ.3 Aristotle so far has given us (1) explicit identification of a fundamental truth about being that all investigators know implicitly, and (2) assertions about its epistemic function as the most firm and certain foundation of knowledge. What he has not done is deal with the issue he left unresolved in the *Analytics*. Since it would be self-contradictory to expect demonstration of the indemonstrable starting points of demonstration, how are these starting points to be justified?

Telling his readers that they could not identify any principle as more worthy of being indemonstrable than PNC (*Met.* Γ.3 1006a11–12) is not in any sense a form of justification. Aristotle reminds us, however, that it shows a "lack of education" to expect there to be demonstration of everything, especially this most secure of all starting points. "Generally speaking, it is impossible for there to be demonstration of everything; for this would proceed to infinity, so in proceeding this way there would be no demonstration.[15] If, however, one ought not to seek a demonstration of certain things, they could not say which principle they deem to be more worthy of being such [i.e., indemonstrable]" (1006a7–12). But in *Metaphysics* Γ.4, after asserting this, he says something surprising: "It is possible to demonstrate in an elenctic manner (*apodeixai elenctikōs*) even about this [denying PNC] that it is impossible, if only the disputant says something" (1006a13–14).

To give an elenctic demonstration is to demonstrate by way of refutation. The practice is discussed in some detail in the work known as *Sophistical Refutations*, a discussion outlined with admirable clarity by Robert Bolton in his 1994 paper "Aristotle's Conception of Metaphysics as a Science." Bolton has shown convincingly that Aristotle is here using, in a very careful and precise way, the technique that is characterized as a form of *peirastic dialectic* in the *Sophistical Refutations*. Aristotle is not, in that case, out to prove any positive thesis. Indeed, from everything he says it is clear he thinks no proof of basic axioms is necessary or possible. Rather, the goal of chapter 4 is to provide a demonstrative refutation of

15. This is, of course, one of the objections to his account of scientific knowledge that he outlines at *APo.* I.3 72b8–16; it is just this objection that requires that there be a non-demonstrative way of knowing first principles, the issue he faces in *APo.* II.19.

PNC-deniers, those who state that it is possible for the same attribute to belong and not to belong to the same thing at the same time and in the same respect. It is a *dialectical* refutation in the sense that the opponent must say something and agree that it signifies one thing. If this minimal requirement is not fulfilled, one might as well be debating a begonia.[16] Nevertheless it is clear, upon reflection, that the refutation works only because it can be shown that, by committing to rational debate of the sort for which Aristotle's *Topics* provides "rules of engagement," the disputant has tacitly accepted what he claims to be denying—namely, PNC.

This reflection on what the parties to rational debate must tacitly accept—for which Alan Code has coined the apt term "meta-elenchus" (1987, 145)—takes up a good deal of the chapter, in fact. In this way we don't produce a question begging demonstration, but rather, since the PNC-denier is responsible for starting the discussion, we produce a refutation. It is a dialectical practice, and therefore (as its name implies) depends on an opponent making a statement that is capable of being refuted. In this case, however, the opponent need not say much! The only requirement is that he says something significant and that it signify one thing.[17] It will not be necessary for my purposes to go through the argument in detail. Bolton (1994, 334–35) lays out the argument in nine steps; Montgomery Furth (1986, commenting on Code 1986), provides an eight-step version. The argument operates via a denial of a "substitution instance" of PNC and forces the opponent to the conclusion that "it is not possible that the same thing at the same time is a man and not a man" (*Met*. Γ.4 1006b33–34). It is not, therefore, proving the truth of PNC but, rather, refuting its denial by using the techniques developed for doing so and relying on concessions made by the PNC-denier. As Aristotle puts it at one point: "While disowning reason, [the denier] submits to reason" (1006a26).[18] The technique draws out the implications of the disputant making a definite claim about a definite object.[19]

16. In *Metaphysics* Γ.5, Cratylus is reported to have decided "he ought to say nothing, and merely moved his finger" (1010a10–13). Apparently before reaching that decision, he stated that Heraclitus's claim that it is not possible to step into the same river twice was mistaken, since it is not possible to do so even once (a13–15).

17. For the detailed ways in which Aristotle here follows the strictures of proof by refutation discussed in *Sophistical Refutations*, see Bolton 1994, 335–36. As one example, Aristotle acknowledges that it would be illegitimate to insist that our opponent say that something either is or is not the case, since that would be a case of begging the question.

18. And see Lear (1980, 103–4).

19. This way of understanding the argument goes back at least to Alexander of Aphrodesias in his commentary on Aristotle *Metaphysics* (Hayduck, ed., 1891, 265.1–267.7); and

The method of validation that is developed in these chapters is redeployed for the Principle of Excluded Middle in chapters 7 and 8. In both cases, the validation of these principles is accomplished by showing that any attempt to articulate a denial of them is self-refuting.

The nature of this demonstration by refutation raises an interesting question about the intended audience of this text—and a passage in chapter 5 is helpful in thinking about that question. The context is a question addressed while discussing the consequences of Protagorean relativism. Aristotle distinguishes between the strategies to be used in arguing with two different sorts of opponent. "But the same mode of argument is not applicable to all; for some need persuasion and some need compulsion. For the ignorance of those who have accepted these ideas out of perplexity is easily remedied; for with them the reply is directed not to a stated position (*logos*) but to reasoning.[20] But the remedy for those who speak for the sake of a stated position is a refutation (*elenchos*) in terms of their statement and their words" (*Met.* Γ.5 1009a16–22).

Demonstration by refutation is not to be used against those who, by drawing faulty conclusions from puzzles about perception, conclude that things can be A and not-A at the same time. Much of Aristotle's chapters 5 and 6 is devoted to resolving such perplexities. Demonstration by refutation is to be used to force those who begin by denying PNC to conclude that their own *logos* commits them to it. In this way you establish its axiomatic status as the firmest of all starting points of demonstration.

But is this a genuine form of non-demonstrative validation for the principles?[21] No: it is an instance of what is sometimes referred to as reaffirmation through denial: any attempt to deny PNC or PEM must, in the end, affirm them. It establishes that PNC is axiomatic. As John Galt puts the point in *Atlas Shrugged*: "An axiom is a proposition that defeats its opponents by the fact that they have to accept it and use it in the process of any attempt to deny it" (*Atlas* 1040). That passage from Galt's speech will serve as a fitting segue into a discussion of Ayn Rand on axioms. We

is developed in Code (1986, 1987). A very different understanding of the argument and its relationship to that of *Posterior Analytics* can be found in Irwin (1988a, 179–98). Irwin's position is among the targets of Bolton (1994). Some interesting objections to Code 1986 can be found in Cohen 1986.

20. Out of context, of course, *logos* can mean many things; but here the context is elenctic disputation, for which one forces an opponent to take up a stated position on a topic.

21. I'm using the term "validation" in a way that has become standard in Objectivist literature. See Peikoff 1991, 8, quoted in my discussion of Rand on the question of validation, below.

will return to the question of validation when we compare the treatment of axioms in Aristotle and Rand, in the third section of this paper.[22]

Ayn Rand on Axiomatic Concepts

In notes in a philosophical notebook dated May 15, 1934, written when she was twenty-nine years old, Ayn Rand was reacting to José Ortega y Gasset's *The Revolt of the Masses*.[23] Very quickly—in a matter of two pages—her reactions move to broader questions of philosophical method, and specifically to questions of how to understand the relationship of philosophical abstractions to concretes. She complains about philosophers acting like mathematicians who are doing algebra as if they had forgotten its relationship to arithmetic—an interesting analogy in light of her theory of concepts, developed much later, in which she describes (proper) conceptual awareness as "the algebra of cognition" (*ITOE* 18). She pauses to reflect on these notes and expresses a sort of dissatisfaction with them but sees the importance of axioms for any area of knowledge, including philosophy: "[W]hat will [ultimately] come out of this is an arrangement of the whole in a logical system, proceeding from a few axioms in a succession of logical theorems. The axioms will be necessary—even mathematics has them—[because] you can't build something on nothing. The end result will be my 'Mathematics of Philosophy'" (*Journals* 72).

Taken out of context, these words have an almost Spinozistic ring. However, the context is critical—in these notes, Rand is expressing concerns about philosophers who act as if abstractions have no connection to the world of concrete particulars, what she would later refer to as "floating abstractions." To avoid this, the axioms she has in mind will need to be grounded in perception in some way. What is most important here, given her later understanding of axioms, is the early recognition of the epistemological necessity of axioms: you cannot build something on nothing; philosophy, like mathematics, needs an axiomatic foundation.

22. In anticipation of the discussion in section 4, the question of the validation of axioms in Ayn Rand, I remind the reader that Aristotle repeats a number of times that we come to know either by induction or by demonstration (*APo.* I.18ff., *APr.* II.23 68b13–14), and that we come to know first principles by induction (*APo.* II.19 100a4–5, *APr.* II.24 68b30–37). No exception is ever suggested for PNC or PEM. Since coming to know them by demonstration is explicitly ruled out, it would seem to commit Aristotle to the view that we come to know metaphysical axioms by induction—but he never explicitly says this, let alone explains how this would work.

23. The material I am using can be found in *Journals of Ayn Rand*, ed. David Harriman. The entirety of her journals are in the Archives at the Ayn Rand Institute.

At this stage of her philosophical development, however, she seems pulled in competing directions for that foundation. On the one hand, she rejects philosophical abstractions that cannot be shown to be based on and applicable to the concretes given to us in perception.[24] On the other hand, she sees that her model of logically ordered knowledge, mathematics, is based on highly abstract axioms upon which its proofs depend. The philosophical account of axioms presented in *Introduction to Objectivist Epistemology*, as we will see, integrates these two apparently conflicting insights.

The Question of Reference

She begins the *Introduction to Objectivist Epistemology* (*ITOE*), published initially in 1966 as a series of articles and then as a monograph, by declaring that "the issue of concepts . . . is philosophy's central issue." In the same paragraph she translates her early concern about the relation of abstractions to concretes into a question about concepts: "concepts are abstractions or universals, and everything that man perceives is particular, concrete. What is the relationship between abstractions and concretes? To what precisely do concepts refer in reality?" (*ITOE* 1). Concepts are abstractions; they refer to an unlimited number of similar entities or attributes.

Reality, however, consists of concrete particulars—the question of the connection of abstractions to concretes is translated into a question about conceptual reference. This starting point has two consequences for her view of axioms. First, in *ITOE* she presents a theory of concepts, and in that context she devotes a chapter to what she refers to as "axiomatic concepts" (*ITOE* 55–61). Second, a central concern in that chapter is to show how these concepts, the most abstract of all concepts, are related to the perceptually given. The primary axiomatic concepts are existence, identity and consciousness.[25] Their axiomatic status derives from their referents. "The units of the concepts 'existence' and 'identity' are every entity, attribute, action, event or phenomenon (including consciousness) that ex-

24. Using an extended metaphor that runs through this entry, where philosophical abstractions are thought of as algebraic and concretes as arithmetic, she worries that "in the field of philosophy today there is this tendency of considering the algebraic formula as final, and therefore philosophy has no practical significance or application" (Journals 71–72).

25. Looking forward to our comparison of Rand's conception of axioms with Aristotle's: two of these concepts (existence and identity) have a clear relationship to two concepts that are central to Aristotle's metaphysics—(1) 'being' and (2) 'what it is to be' or 'essence.' One of them, however (consciousness), does not. Its inclusion is of great historical significance and is the key to a number of Rand's philosophical innovations.

ists, has ever existed or will ever exist. The units of the concept 'consciousness' are every state or process of awareness that one experiences, has ever experienced or will ever experience" (56). To regard entities as "units" in Rand's sense is to focus on one or more of their attributes, to isolate them in virtue of how they are different from other entities, and then to integrate them in virtue of their similarities with respect to the attributes in focus. "*The ability to regard entities as units is man's distinctive method of cognition, which other species are unable to follow*" (6; italics in original).

Axiomatic concepts identify aspects of reality implicit in all knowledge—facts that are fundamentally given and directly perceived (*ITOE* 55). One can unpack these concepts in propositional form only by way of repetition: What is, *is*; it is *what it is*; and consciousness is the awareness of and the means of *identifying* what is. These most fundamental of all truths are incapable of proof, for all proof rests on them. Any attempt to establish them by a proof, Rand argues, would be self-contradictory (55). What one *can* prove is precisely that they are axiomatic, by showing that they are presupposed, even in attempts to deny them. On this point she and Aristotle are in complete agreement.

Their axiomatic status, then, derives from the aspects of reality they identify being self-evident in perception. But if their content is perceptually given, why are these concepts so abstract? This is, perhaps, their most puzzling feature. "Their peculiarity lies in the fact that they are perceived or experienced directly, but grasped conceptually. They are implicit in every state of awareness, from the first sensation to the first percept to the sum of all concepts. . . . [t]hese facts are contained in any single state of awareness; but what is added by subsequent knowledge is the epistemological need to identify them consciously and self-consciously" (*ITOE* 55–56).[26] Prior to the explicit formation of axiomatic concepts, Rand argues, they are implicit in every act of awareness. Underscoring their importance, this is a point she makes on the first page of the first chapter of *ITOE*:

> The building-block of man's knowledge is the concept of an "*existent*"—of something that exists, be it a thing, an attribute or an action. Since it is a concept, man cannot grasp it *explicitly* until he has reached the conceptual stage. But it is implicit in every percept (to perceive a thing is to perceive that it exists) and man grasps it implicitly on the perceptual level—i.e., he grasps the constituents of the concept "existent," the data which

26. This is her answer to the concerns she expressed in her philosophical notebook dated May 15, 1934. See *Journals*, 70–73.

> are later to be integrated by that concept. It is this implicit knowledge that permits his consciousness to develop further. (5–6)[27]

The above description of the units of these concepts poses a prima facie problem, given the theory of concept-formation presented in the following chapters. For this theory argues that concepts are formed by integrating their referents through a process that crucially depends on differentiating those units from the units of other concepts that share a "conceptual common denominator" with them.[28] For example, one forms the concept "plate" by differentiating plates from other sorts of tableware (bowls, cups, saucers, etc.), according to their distinctive range of commensurable features of shape and function, and then integrating the plate-units into a single concept by omitting the measurable differences among them. Axiomatic concepts, however, cannot be formed by such a process. Rand faces this issue directly in *ITOE*, in the chapter devoted to axiomatic concepts. "Since axiomatic concepts are not formed by differentiating one group of existents from others, but represent an integration of all existents, they have no Conceptual Common Denominator with anything else. They have no contraries, no alternatives" (*ITOE* 58). What, then, is the nature of the distinctive cognitive process involved in explicitly grasping axiomatic concepts? The full answer to that question has two aspects. Part of the answer has to do with the peculiar nature of the abstraction involved in their case: "It [the abstraction involved in forming axiomatic concepts] is not the abstraction of an attribute from a group of existents, but of a basic fact from all facts. Existence and identity are *not attributes* of existents, they *are* the existents. Consciousness is an attribute of certain living entities, but it is not an attribute of a given state of awareness, it *is* that state" (56). From an epistemological standpoint, as

27. On the relationship between the concepts "existent" and "existence," see *ITOE* appendix, 241. The following personal anecdote may be of help in concretizing this move from implicit to explicit conceptual identification of axioms. I was driving my five-year-old daughter to kindergarten and (it began as a game) she said something like "a tree is a tree," to which I responded "and a car is a car," to which she replied "and a house is a house." After another minute or so of this, she paused—and then said, with a big smile on her face, "and everything is just what it is." At which point I realized that, for my daughter, the Principle of Identity had just gone from implicit to explicit status.

28. "Observe the multiple role of measurements in the process of concept-formation, in both of its two essential parts: differentiation and integration. Concepts cannot be formed at random. All concepts are formed by first differentiating two or more existents from other existents. All conceptual differentiations are made in terms of commensurable characteristics (i.e., characteristics possessing a common unit of measurement)" (*ITOE* 13).

she goes on to stress, the process of abstraction in the case of axiomatic concepts involves selective focus on metaphysical fundamentals. To conceptualize "existence" and "identity" (which Rand considers corollaries) is an act of abstraction because it requires a selective focus, not on an entity's size, color, shape, behavior, and so on, but on the fact that any object of awareness *is*, and is something *specific*.[29] "Consciousness" does not refer to a fundamental feature of "beings qua being"—it refers to a feature of a restricted class of beings. It is, however, equally fundamental to *knowledge*. Axiomatic concepts, then, refer to aspects of reality qua fundamental to every act of knowing. Their axiomatic status derives from the fact that the aspects of reality they identify are implicit in every act of awareness.

There is, moreover, an act of measurement-omission involved in the formation of axiomatic concepts. "The measurements omitted from axiomatic concepts are all the measurements of all the existents they subsume; what is retained, metaphysically, is only a fundamental fact; what is retained, epistemologically, is only one category of measurement, omitting its particulars: *time*—i.e., the fundamental fact is retained independent of any particular moment of awareness" (*ITOE* 56). In the discussions of this chapter in the Epistemology Workshops transcripts, selected portions of which are provided as an appendix to *ITOE*, one of the participants pressed Rand to elaborate on the distinction being made here between what is retained metaphysically and epistemologically. Her reply is helpful; it relates both to the topic of the referents of axiomatic concepts and to what Rand would characterize as their psychological and epistemological functions. That aspect of her reply will be taken up in the next section below. At this point, here is the relevant part of her response: "What I mean here by 'metaphysically' is: in reality, in existence—that is, focusing on the entities subsumed under the concept. By 'epistemologically' here I mean psychologically. In the process of cognition, what type of measurement do you retain when you deal with a concept that includes every existent?" (*ITOE* 256). In the process of forming an abstraction, as Rand understands it, one omits the exact measurements of the attribute in question but retains the category of measurement—for example, in forming the concept "length," one omits the different lengths of whatever objects one is comparing in forming the concept (pencils, rulers, fingers,

29. Why it is important to attend to those aspects of what one is aware of is the topic of the next section.

legs) but retains that attribute.[30] But what is of especial interest in the case of axiomatic concepts is the distinction between two different categories of what is retained in forming them: metaphysically, what is retained is simply the fact that the object of awareness *is* (something *specific*); epistemologically, what is retained is the category time. In order to delve into why Rand distinguishes these two aspects of abstraction and why, in particular, it is *temporal* measurements that are omitted psychologically, we need to turn to Rand's understanding of the function(s) of axiomatic concepts.

The Question of Function(s)

Early in the presentation of her account of concept-formation, Rand argues that *implicit* knowledge of these concepts—that is, awareness of the data that will later be integrated into these concepts—suffices for human beings to develop cognitively to the conceptual level (*ITOE* 6). Why, then, do we need to form these concepts at all—why must we make that knowledge explicit? Conveniently, for my purposes, Rand introduces the question of why it is important to identify axioms in conceptual terms immediately after summarizing their distinctive referential status: "Their peculiarity lies in the fact that they are perceived or experienced directly, but grasped conceptually. They are implicit in every state of awareness, from the first sensation to the first percept to the sum of all concepts. . . . [T]hese facts are contained in any single state of awareness; but what is added by subsequent knowledge is *the epistemological need* to identify them consciously and self-consciously" (55–56; emphasis added).

Before we explore the question of why subsequent knowledge creates an epistemological need to grasp the axiomatic concepts explicitly, I want to draw attention to a distinctive feature of Ayn Rand's philosophical method operative in this passage. Rather than assuming that explicit conceptual recognition of axioms is of value, she first asks, and then answers, the question "Why is explicit recognition of axioms necessary?" So too in her metaethics, her starting point is the question of whether human beings need a code of values—that is, an explicitly codified morality: "Let me stress this. The first question is not: What particular code of values should man accept? The first question is*: Does man need* values at

30. Rand spells out the process as she conceives it by imagining someone describing the process as they are experiencing it: "Length must exist in *some* quantity, but may exist in *any* quantity. I shall identify as 'length' that attribute of any existent possessing it which can be quantitatively related to a unit of length, without specifying the quantity" (*ITOE* 11).